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Switched Bias Proportional Navigation for Homing Guidance Against Highly Maneuvering Targets

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A new form of the Proportional Navigation (PN) guidance law for short-range homing missiles is proposed. Named the Switched Bias Proportional Navigation (SBPN) law, it is derived by invoking sliding-mode control theory and is structured around the basic PN, with an additive switched bias term. This additional term depends only on the polarity of the line-of-sight rate, which is readily available with a seeker. It is shown that the bias term acts as an estimate of the target acceleration and other unmodeled dynamics. An adaptive procedure is suggested to select the gain of this term, which results in improved performance. The SBPN is almost as simple to implement as the PN law itself, as it does not require any additional information related to the engagement, in the form of either measurements or estimates. Simulation results show that the acceleration profiles of SBPN closely follow those of augmented PN guidance law, after a short initial transient. They further demonstrate the robustness of the proposed SBPN in the presence of missile velocity variation.

I. Introduction

GUIDANCE of short-range homing missiles has been a topic of intensive research for over four decades. Many of the popular guidance laws cited in open literature[1–6] are based on Proportional Navigation (PN) and its variants. The PN law seeks to null the Line-Of-Sight (LOS) rate against nonmaneuvering targets by making the missile heading rate proportional to the LOS rate while closing in on range. In fact, PN has been shown to be optimal[7] for linearized engagement equations, with constant-speed missile and nonmaneuvering target models, and for certain specific forms of the quadratic performance index. Even though PN performs reasonably well in a wide range of engagement geometries, its performance sharply degrades in the presence of rapidly maneuvering targets and large off-boresight angle missile launches.

Several modifications have been suggested in order to improve the performance of the basic PN in the presence of large heading errors and smart target maneuvers. These modifications are generally in the form of a bias added to the PN to compensate for such effects. Brainin and McGhee[8] superimpose a constant bias to the measured LOS rate before calculating the commanded acceleration using the PN law. Assuming constant normal target acceleration, by linearization of the engagement geometry around the normal collision course and using small-angle assumptions, they express the bias term as a function of target acceleration and the initial heading error. Using a numerical approach, they then optimize this bias term to minimize the control effort. In an extension of Ref. 8, Shukla and Mahapatra[9] obtain an analytical expression for this bias term.

In a significantly different approach, Speyer[10] uses linear exponential, Gaussian control theory to derive the optimal biased PN law, which explicitly includes first-order missile dynamics and a Gauss-Markov process-modeled target acceleration. This guidance law is essentially a PN law with time-varying navigation gain and additional bias terms, which are functions of the target and missile accelerations. Nazaroff[11] considers time-varying missile velocity and uses optimal control theory to obtain a guidance law similar to PN but with additional terms to compensate for the missile velocity variation, target acceleration and jerk.

Several other variants of PN similar to those mentioned above exist in the literature,[12–15] all of which claim to improve the performance of the PN guidance law. A survey of guidance laws can be found in Ref. 5 and, more recently, in Cloutier et al.[6] The main disadvantage of these variants, from the implementation point of view, is that they require explicit knowledge of target acceleration, missile velocity variation, and missile lateral acceleration in order to be implemented. This necessitates two sets of additional requirements: a) augmentation of sensors to measure forward acceleration and b) estimation of target acceleration by suitable analytical methods. This makes the guidance law more complex, besides reducing the cost-effectiveness of the PN.

A large number of target acceleration estimation methods available in the literature[6] are generally based on the Kalman filter and the extended Kalman filter structures. The problem associated with these approaches is that assumptions have to be made about the target acceleration dynamics, and when these assumptions are violated, they result in modeling errors that may lead to divergence of the estimation process or in poor estimates of the target acceleration. Such estimates may not serve any useful purpose when used in the guidance law.

A new form of guidance law based on Sliding-Mode Control (SMC) theory, called Switched Bias Proportional Navigation (SBPN), is proposed here to overcome the above cited limitations. Like the other variants of PN, this is structured around the basic PN, with an additional bias term to compensate for target acceleration and other unmodeled dynamics. In SBPN, this additional bias term is a function of the LOS rate, which is readily available with a homing seeker. The main advantage of the present approach is that it is almost as simple to implement as the PN law itself. Simulation results show SBPN to be robust against a wide class of intelligent target maneuvers.

II. Engagement Geometry

In this section, the engagement geometry that describes the missile target relative kinematics is presented. For the purpose of simplicity, only a two-dimensional engagement model is considered. The results can be easily extended to the three-dimensional case. Further, in modeling the engagement dynamics, we assume the following:

A1) Missile and target are treated as point mass models.

A2) Missile and target velocities v_m, v_t are constant and the missile has a speed advantage over the target, i.e., $v_m \geq v_t$.

A3) Autopilot and seeker loop dynamics are fast enough to be neglected when compared to the overall guidance loop behavior.

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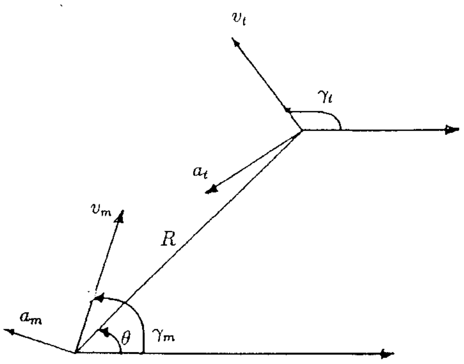


Fig. 1 Two-dimensional engagement geometry.

A4) Target acceleration, a_t is assumed to be bounded with a known, bound on its magnitude, i.e., $|a_t| < \alpha$, where α is a known positive constant.

Under the above assumptions, the two dimensional engagement model (Fig. 1) can be represented mathematically by the following differential equations:

$$\dot{R} = v_t \cos(\gamma_t - \theta) - v_m \cos(\gamma_m - \theta) \tag{1}$$

$$R\dot{\theta} = v_t \sin(\gamma_t - \theta) - v_m \sin(\gamma_m - \theta) \tag{2}$$

$$\dot{\gamma_m} = a_m / v_m = a_c / v_m \tag{3}$$

$$\dot{\gamma_t} = a_t / v_t \tag{4}$$

where

$R, \dot{R}$ = relative range and range rate to target
γ_m, γ_t = flight path angles of missile and target
$\theta, \dot{\theta}$ = LOS angle and its rate
a_m, a_c = lateral and commanded accelerations of missile

Assumptions A1–A3 are generally made in the design of short-range homing guidance laws. Assumption A4 is not restrictive in the sense that it only requires the knowledge of target acceleration bound and not its actual time history. Generally, such information is available for the guidance law designer.

III. Guidance Law Derivation

The main objective here is to derive a simple and easy-to-implement guidance law that offers robustness against a variety of target maneuvers and off-nominal conditions, encountered in the terminal phase of a short-range homing missile. SMC systems based on Variable Structure Control (VSC) theory developed by Utkin,[16] are distinguished by their robustness properties against a class of bounded disturbances. In recent years, SMC theory has found widespread usage in such diverse areas as control of robots, aircraft, large space structures, etc., but does not seem to have been applied in the specific context of missile guidance.

In general, VSC system design can be broken into two phases. The first phase entails the construction of a switching surface so that the system restricted to this surface produces the desired behavior. The next step is to choose a control that will drive the system trajectories onto the switching surface and constrain them to slide along this surface for all subsequent time. Since the desired surface is chosen such that it is independent of the external disturbances, robustness can be achieved.

To apply VSC theory to the guidance law design, a switching surface that represents the desired system dynamics is to be chosen. The selection of the switching surface is crucial because the structure of the guidance law and its robustness properties are very much dependent on it. It will be shown that the choice of switching surface

$$S = \dot{\theta} \tag{5}$$

will result in a guidance law that is simple and easy-to-implement and is robust against a wide variety of target maneuvers. The basic idea behind the selection of the above switching surface is to null the LOS rate. Suppose that we are able to achieve $\dot{\theta} \equiv 0$ by a suitable choice of control. Then Eq. (5) together with assumption A2 implies that $\dot{R} < 0$, thus guaranteeing interception. Note that nulling the LOS rate is also the objective of the basic PN law and PN achieves it for nonmaneuvering targets and constant-speed missiles, by making the missile heading rate proportional to the LOS rate. It will be interesting to examine how a guidance law based on VSC theory differs from the classical PN.

Having defined the switching surface, the next step is to design a control law that will guarantee the attractivity of the surface $S = 0$ and also the sliding along $S = 0$. To achieve this, we construct the following Lyapunov function:

$$V = \tfrac{1}{2} S^2 \tag{6}$$

Obviously, this function is positive definite. A sufficient condition in order to guarantee the attractiveness of $S = 0$ is to ensure $\dot{V} = \dot{S}\, S < 0$ for $S \neq 0$:

$$\dot{V} = \frac{\dot{\theta}}{R}[-2\dot{R}\dot{\theta} - a_m \cos(\gamma_m - \theta) + a_t \cos(\gamma_t - \theta)] \tag{7}$$

A choice of the control a_c that will ensure $\dot{V} < 0$ is

$$a_c = \frac{1}{\cos(\gamma_m - \theta)}[-2\dot{R}\dot{\theta} + K\dot{\theta} + W\,\mathrm{Sgn}(\dot{\theta})] \tag{8}$$

where

$$\mathrm{Sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$W \geq \alpha + \mu$, and μ, $K > 0$. Substituting Eq. (8) into (7), we obtain

$$\dot{V} = \frac{\dot{\theta}}{R}[-K\dot{\theta} - W\,\mathrm{Sgn}(\dot{\theta}) + a_t \cos(\gamma_t - \theta)]$$

$$\leq \frac{\dot{\theta}}{R}[-K\dot{\theta} - \mu\,\mathrm{Sgn}(\dot{\theta})] < 0 \tag{9}$$

Thus the choice of a_c given by Eq. (8) guarantees attractiveness of the switching surface $S = 0$. A feasible choice for the gains K and μ is next presented. Suppose that $K = -K'\dot{R}$ and $\mu = -K'\rho\dot{R}$. From Eq. (7), this implies that, when $\dot{\theta} \geq 0$,

$$\ddot{\theta} \leq \frac{1}{R}[K'\dot{R}\dot{\theta} + K'\rho\dot{R}\,\mathrm{Sgn}(\dot{\theta})] < 0$$

$$\Rightarrow \ddot{\theta}/(\dot{\theta} + \rho) \leq K'\dot{R}/R \tag{10}$$

By integrating both sides, we obtain

$$\dot{\theta} + \rho \leq [\dot{\theta}(0) + \rho][\dot{R}/R(0)]^{1/K'}$$

A similar expression can be derived for $\dot{\theta} < 0$, and it can be easily shown that $\dot{\theta} = 0$ will be reached at a relative range

$$R \leq R(0)\{\rho/[|\dot{\theta}(0)| + \rho]\}^{1/K'} \tag{11}$$

where $K' > 0$ and $\dot{\theta}(0)$ and $R(0)$ are the initial values of the LOS rate and range, respectively. With this choice of K and μ, the guidance law (8) can be written as:

$$a_c = \frac{1}{\cos(\gamma_m - \theta)}[-N\dot{R}\dot{\theta} + W\,\mathrm{Sgn}(\dot{\theta})] \tag{12}$$

where $W \geq \alpha - K'\rho\dot{R}$ and $N = K' + 2$. It is assumed here that the initial conditions are such that the guidance law (8) guarantees $\dot{R} < 0$ during the reaching phase. Following similar lines of argument as in Ref. 17, one can easily derive the initial conditions, which will result in $\dot{R} < 0$, while using the guidance law (12). Note that this choice of a_c assumes the knowledge of $\dot{R}$, $\dot{\theta}$, and $(\gamma_m - \theta)$. The first two quantities are generally available for a missile equipped with an active seeker. On the assumption of small boresight error and angle of attack, the angle $(\gamma_m - \theta)$ can be approximated with the gimbal angle, which can be measured. The new guidance law given by Eq. (12) can be regarded as the PN guidance law with time-varying navigation gain, due to the presence of the $\cos(\gamma_m - \theta)$ term and with a switched bias term. This law can be viewed as a form of the Modified Proportional Navigation (MPN) proposed by Ha et al.[17] for the case of active homing, with the acceleration estimate term in the MPN being replaced by the Sgn term. The main advantage of this guidance law over MPN is that it does not require any explicit target maneuver estimation.

Significance of the Bias Term

The significance of the additive term can be explained as follows. Suppose that the control law is able to guarantee the sliding condition and hence the $\dot{\theta}$ trajectories are sliding along $S \equiv 0$ in the steady state. However, due to the presence of disturbances like target acceleration, it is not possible to achieve $\dot{\theta} = 0$ exactly. Instead, $\dot{\theta}$ will remain close to zero. In the neighborhood of the sliding surface $\ddot{\theta} \approx 0$ implies

$$-K'\dot{R}\dot{\theta} - W\,\mathrm{Sgn}(\dot{\theta}) + a_t\cos(\gamma_t - \theta) \approx 0 \tag{13}$$

that is,

$$a_t\cos(\gamma_t - \theta) \approx -K'\dot{R}\dot{\theta} + W\,\mathrm{Sgn}(\dot{\theta}) \tag{14}$$

When the sliding condition is satisfied and the system is sliding along the surface $\dot{\theta} \approx 0$, the additive terms in Eq. (14) together act as a target acceleration estimate. In other words, when the system is in the sliding-mode steady state, the guidance law (12) acts as an Augmented Proportional Navigation (APN) guidance law with a navigation gain of $2/\cos(\gamma_m - \theta)$.

Fixed Navigation Ratio

The guidance law given by Eq. (12) can be reduced to the familiar PN form by dropping the $\cos(\gamma_m - \theta)$ term with the assumption that it is approximately equal to 1. It can be easily shown that, in order to guarantee the attractivity of the sliding surface, the switching gain has to be modified to:

$$W \geq \frac{\alpha + \mu + \beta|a_m|}{\cos(\gamma_m - \theta)} \tag{15}$$

where β is a constant such that $|1 - \cos(\gamma_m - \theta)| \leq \beta$. In order to guarantee the reaching condition, it requires that $\cos(\gamma_m - \theta)$ must be positive, which implies that $(\gamma_m - \theta)$ should be kept acute throughout the engagement. In practice, this condition is usually satisfied because of the mechanical constraints imposed. In other words, replacing $\cos(\gamma_m - \theta)$ by 1 necessitates a higher switching gain in order to satisfy the reaching condition.

From the above analysis, it is clear that the parameters K', α, μ, and β play an important role, and proper selection of their values is critical for the performance of SBPN. The following section provides some guidelines for the selection of these parameters.

IV. Selection of Guidance Law Parameters

Selection of α

The switching gain α represents the uncertainty bound regarding the target acceleration. There exist other unmodeled dynamics like missile velocity variations, neglected seeker and track loop dynamics, etc. The sliding-mode control theory provides a way of coping with these uncertainties, if they can be bounded and included in the uncertainty bound α, which until now has been used to represent the target acceleration bound alone. The problem associated with this approach is that, in general, it is difficult to find an exact bound for these uncertainties. Although a conservative bound may require excessive control effort in the reaching phase, an underestimated bound may violate the reaching condition and hence can lead to loss of stability.

In order to eliminate this problem, α is replaced with $\hat{\alpha}$ in Eq. (12) where $\hat{\alpha}(t)$ is given by

$$\hat{\alpha}(t) = \alpha_0 + \alpha_1 \int_0^t |R\dot{\theta}|\,\mathrm{d}\zeta \tag{16}$$

where α_0 and α_1 are some positive numbers. The motive behind this selection is explained herein.

Consider a Lyapunov-like function

$$V = \tfrac{1}{2}R^2S^2 + \frac{1}{2\alpha_1}\tilde{\alpha}^2 \tag{17}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}(t)$. This particular function is chosen to simplify the selection of $\hat{\alpha}(t)$. Taking the total time derivative of V and using Eqs. (8) and (16), we get

$$\dot{V} = R\dot{\theta}[\dot{R}\dot{\theta} - K\dot{\theta} + a_t\cos(\gamma_t - \theta) - \hat{\alpha}\,\mathrm{Sgn}(\dot{\theta})] + \frac{1}{\alpha_1}\tilde{\alpha}\dot{\tilde{\alpha}} - \mu R\dot{\theta}\,\mathrm{Sgn}(\dot{\theta}) \tag{18}$$

$$\leq (\dot{R} - K)R\dot{\theta}^2 + |R\dot{\theta}|\tilde{\alpha} + \frac{1}{\alpha_1}\tilde{\alpha}\dot{\tilde{\alpha}} - \mu R\dot{\theta}\,\mathrm{Sgn}(\dot{\theta}) \tag{19}$$

If we can force the above term to be negative definite, we can guarantee the stability of the system. If we assume that $\dot{R} < 0$ is maintained with this modification of the guidance law, the first and fourth terms are negative definite by choice, and the second and third terms cannot always be made negative definite by any choice of a_m. But the choice of

$$\dot{\tilde{\alpha}} = -\alpha_1|R\dot{\theta}| \tag{20}$$

that is,

$$\hat{\alpha}(t) = \alpha_0 + \alpha_1\int_0^t |R\dot{\theta}|\,\mathrm{d}\zeta \tag{21}$$

where α_0 is the initial value of $\hat{\alpha}$, leads to

$$\dot{V} < (\dot{R} - K)R\dot{\theta}^2 - \mu R\dot{\theta}\,\mathrm{Sgn}(\dot{\theta}) < 0 \tag{22}$$

which ensures stability of $\dot{\theta}$. However, the above choice of $\hat{\alpha}$ does not guarantee that $\hat{\alpha}$ will converge to the true value of α. Nevertheless, it ensures the satisfaction of the reaching condition and hence sliding along $\dot{\theta} = 0$ is guaranteed even in the presence of a mismatched bound. The main advantage of this approach is that instead of using conservative estimates of α, which will result in a large control effort, one can start with a less conservative estimate α_0 and can still maintain the attractivity of the switching surface, even in the case of possibly underestimated initial uncertainty bound, represented by α_0.

With these modifications, the proposed guidance law can be expressed as:

$$\begin{aligned} a_m &= -\frac{N}{\cos(\gamma_m - \theta)}\dot{R}\dot{\theta} + W\,\mathrm{Sgn}(\dot{\theta}) \\ W &\geq \alpha - K'\rho\dot{R} \\ \alpha &= \alpha_0 + \alpha_1\int_0^t |R\dot{\theta}|\,\mathrm{d}\zeta \end{aligned} \tag{23}$$

For the case of fixed navigation ratio, the bound W should be such that

$$W \geq \frac{\alpha - K'\rho\dot{R} + \beta|a_m|}{\cos(\gamma_m - \theta)} \qquad \beta \geq |1 - \cos(\gamma_m - \theta)|$$

where K', ρ, α_0, α_1, and β are some positive real numbers. Note that the above form of SBPN requires range and range rate in addition to LOS rate information.

Selection of Gains

It can be observed from the previous analysis that the disturbance rejection properties of the guidance law are limited to the steady state of the sliding phase and not the reaching phase. In order to guarantee robustness, the system must be quickly attracted to the switching surface and maintained there. Examination of Eq. (11) reveals that the gains N, α_0, α_1, and ρ play an important role in controlling $\dot{\theta}$ trajectories during the reaching phase. Some general guidelines can be formulated for choosing these gains.

As the linear gain N is effective only during the reaching phase, it can be chosen such that it will not result in excessive missile acceleration. For example, assuming that the target acceleration is zero and using the analysis of Ref. 17, one can fix the value of N that guarantees $\dot{R} < 0$ during the reaching phase. In other words, the navigation ratio of SBPN can be chosen to be smaller than that of PN to intercept the same maneuvering target.

The function of the switching gain ρ together with $\hat{\alpha}$ given by Eq. (16) is to force the $\dot{\theta}$ trajectories onto the switching surface and maintain the attractivity during the sliding mode. Theoretically, the larger the ρ, the more robust is the guidance law with respect to the unmodeled dynamics and also the smaller is the reaching time. But in many practical situations, because of the presence of nonidealities, the ideal sliding mode is not guaranteed, but the $\dot{\theta}$ will stay within some boundary layer of the sliding surface. In this case, the presence of Sgn($\cdot$) in the guidance law will lead to high-frequency oscillations (often called chattering), whose amplitude is proportional to the gain of the switching term W. In order to reduce the amplitude of these oscillations, it becomes necessary to choose as small a value of ρ as possible in the sliding mode. The following analysis will explain this.

Suppose that Sgn($\dot{\theta}$) can be approximated by $\dot{\theta}/(|\dot{\theta}|+\delta)$, where δ is a small positive real number. When the system is near the sliding mode, $\dot{\theta} \approx 0$, and assuming that $\delta \gg \dot{\theta}$, the guidance law can be written as

$$a_m = -N\dot{R}\dot{\theta} + (-K'\dot{R}\rho + \hat{\alpha})\dot{\theta}/\delta \tag{24}$$

that is, in the steady-state sliding mode, the proposed guidance law acts as a PN law with a navigation ratio of $N+(K'\rho-\hat{\alpha}/\dot{R})/\delta$, which means that during the reaching phase, the approximated switching term acts as a bias and during the sliding mode it acts as a large linear gain.

From the above analysis, it can be concluded that the gain of the switching term, W should be 1) large enough to result in faster reaching without resulting in excessive acceleration while in the reaching phase and 2) small enough to not result in excessive chattering during the sliding mode. Keeping these conflicting requirements in mind, the gains ρ, α_0, and α_1 can be chosen so as to result in good performance.

Regarding the selection of β, in many practical engagement conditions, $\cos(\gamma_m - \theta)$ would remain close to 1, and hence β can be chosen to be a small positive number close to zero. The simulation studies presented in the next section provide further insight into the choice of specific numerical values for the switching gains.

V. Simulation Results

This section evaluates, through simulation studies, the performance of the proposed SBPN guidance law in the presence of highly maneuvering targets. Specific simulation studies are presented here for two engagement scenarios.

Before proceeding with the simulation studies, the structure of the guidance law (12) is slightly modified mainly from the implementation considerations. One of the underlying assumptions in the design of this guidance law has been that the missile acceleration can be instantaneously switched from one value to another, at will. In practice, however, because of various system nonidealities, switching of the control at a very fast rate will result in chattering of the control signal, which in turn may excite the unmodeled high-frequency dynamics. The chattering can be eliminated by replacing the discontinuous Sgn($\cdot$) with a continuous approximation, like the high gain saturation function. In the present study, an approximation of the form $x/(|x|+\delta)$, where $\delta > 0$ is a small positive number, has been used instead of Sgn(x) in Eq. (12).

Engagement Scenarios Considered

Two different engagement scenarios are considered for the purpose of simulation. In the first case, referred to as Engagement 1, the target is assumed to pull a constant dive turn of 80 m/s^2 throughout the engagement. In the second, denoted as Engagement 2, the target initiates an 80- m/s^2 dive turn at the beginning of interception, switches to level flight after 2 s, and then executes an 80- m/s^2 evasive maneuver after 2 more seconds and continues with it until interception (Fig. 2). In order to impart more realism to the studies, additional features in the form of a first-order missile and target autopilots and seeker dynamics have also been incorporated. Further, in this deterministic study, all the quantities required for the implementation of the guidance laws are assumed to be available without any measurement errors. Table 1 presents the rest of the simulation data used.

Table 1 Simulation data

R	4500 m
v_m	500 m/s
v_t	300 m/s
θ	20 deg
γ_t	140.00 deg
α	80.0
δ	0.0017
N	4
K'	2
ρ	0.01
Heading error	10 deg

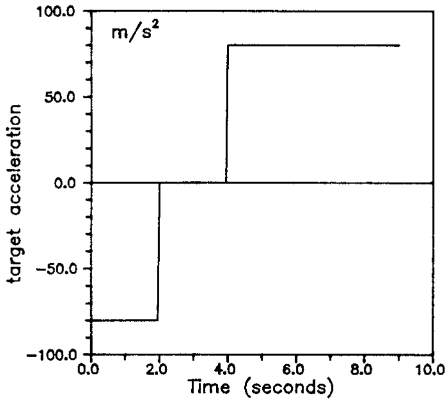


Fig. 2 Target acceleration time history for Engagement 2.

For the purpose of comparison, the following versions of PN and APN laws with their navigation ratios modified to include the $\cos(\gamma_m - \theta)$ term are employed in the simulation studies:

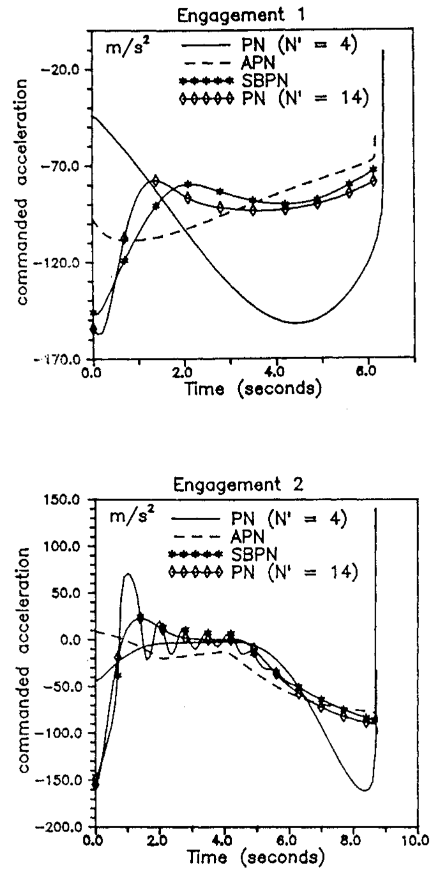
$$a_{\text{cpn}} = -[N'/\cos(\gamma_m - \theta)]\dot{R}\dot{\theta} \tag{25}$$

$$a_{\text{capn}} = -[N'/\cos(\gamma_m - \theta)]\dot{R}\dot{\theta} + a_t\cos(\gamma_t - \theta) \tag{26}$$

Here a_{cpn} and a_{capn} denote the commanded accelerations with PN and APN, respectively. For the purpose of carrying out the comparative study on a common basis, an effective navigation ratio $N' = 4$ has been selected in all the three schemes. Moreover, since the performance of the SBPN guidance law with a fixed and variable navigation ratios has been found to be quite similar, the results for the former have not been presented in this section.

Fixed α Case

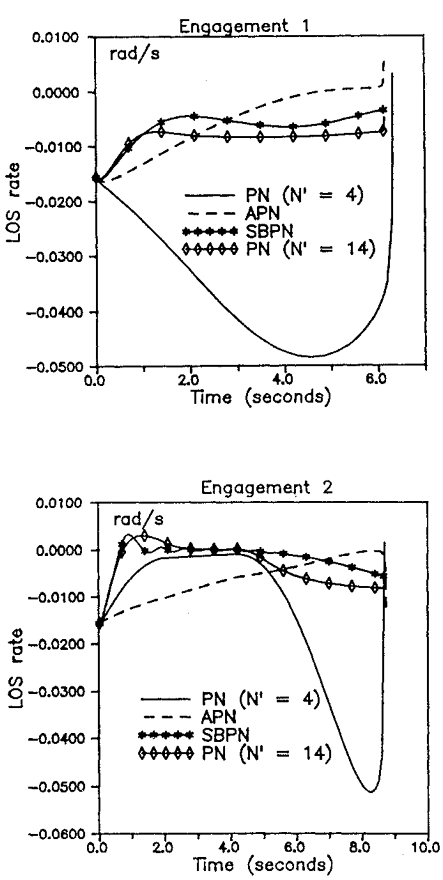
The performance of the SBPN guidance law with a fixed switching gain term α is considered. The acceleration profiles of the missile for the two engagement geometries considered are shown in Fig. 3. It can be seen that whereas PN, with $N' = 4$, requires excessive acceleration, the acceleration profiles of SBPN follow those of APN after a short initial transient. It is interesting to observe that the acceleration profiles of PN with $N' = 14$ resemble those of SBPN in both the engagements considered. Although the LOS rates (Fig. 4) of PN with $N' = 14$, APN, and SBPN remain close to zero after a short initial time, that of PN with $N' = 4$ exhibits a completely different trend. In the sliding mode, i.e., when the LOS rate is nulled, that the additional switching term in SBPN indeed acts as the estimate of the target acceleration component normal to the LOS is demonstrated in Fig. 5. The oscillatory nature of the commanded



a)

b)

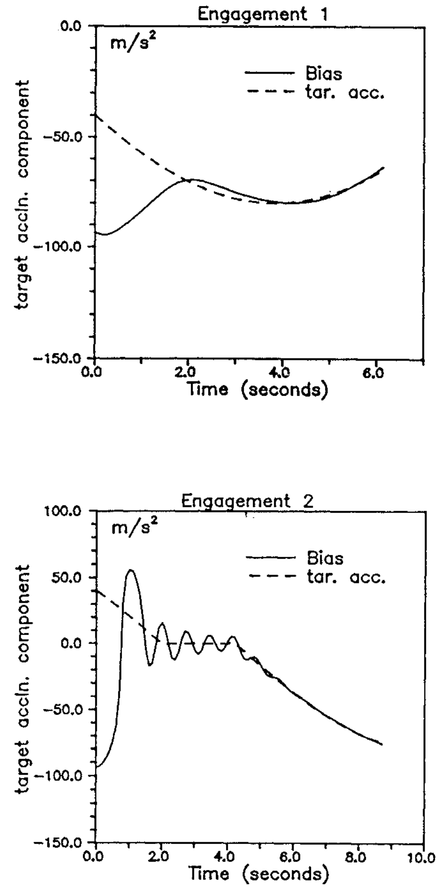
Fig. 3 Commanded acceleration profiles.



a)

b)

Fig. 4 LOS rate profiles.



a)

b)

Fig. 5 Target acceleration and switched bias profiles.

acceleration and switched bias profiles for the case of SBPN in Engagement 2, during the absence of target maneuver, is due to the large value of α chosen.

Table 2 presents the miss distance and flight time data for the two engagements considered. The time taken for interception is approximately the same for SBPN, APN, and PN with $N' = 14$, whereas for PN with $N' = 4$ it takes longer. Further, the PN causes larger miss distance values, when compared with the other three guidance laws considered. The miss distance values with SBPN, APN, and PN with $N' = 14$ are all of the same order.

Adaptive α Case

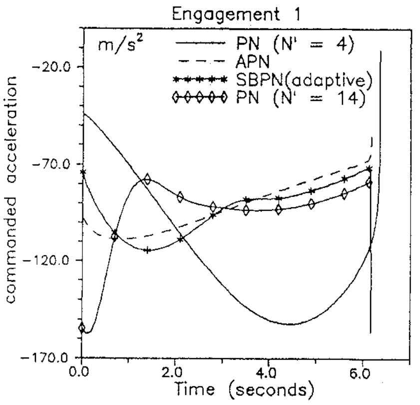
In order to illustrate the use of the adaptive bound estimation, the two engagement scenarios are again simulated using $\hat{\alpha}$ given by Eq. (16), instead of the fixed α value in the guidance law (12). The parameter α_0 is set to 20 and α_1 is fixed at 0.5. Figure 6 shows the acceleration history of SBPN in comparison with APN, PN with $N' = 14$, and the fixed α case considered in the previous study. The adaptive selection of α reduces the magnitude of the initial transient and eliminates the chattering associated with Engagement 2. The inference is that the SBPN is different from that of a PN with higher value of N', due to its adaptive nature. The time history of the adaptive parameter $\hat{\alpha}$ is given in Fig. 7. Note that $\hat{\alpha}$ increases mostly during the first 3 s and then remains steady, implying that, when satisfactory performance is achieved, the adaptation stops. This scheme results in miss distance values of 0.52 m and 0.24 m, respectively, which are of the same order as that of the nonadaptive case. It may further be inferred that the proposed adaptation scheme helps in reducing the magnitude of the initial transient but leaves the miss distance values unaffected.

Effect of Missile Velocity Variation

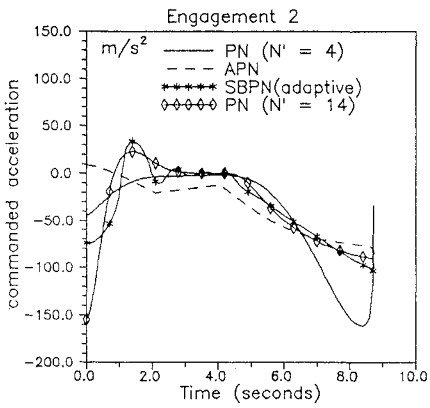
In order to demonstrate the robustness of the SBPN, the missile velocity has been assumed to decrease at a rate of 20 m/s^2. The

Table 2 Miss distances and flight times

	Engagement 1		Engagement 2	
Guidance law	Miss distance, mtrs	Flight time, s	Miss distance, mtrs	Flight time, s
PN ($N' = 4$)	4.6756	6.351	0.8685	8.736
APN	0.485	6.167	0.4152	8.751
SBPN	0.133	6.145	0.1969	8.701
PN ($N' = 14$)	0.461	6.155	0.361	8.704



a)



b)

Fig. 6 Commanded acceleration profiles.

simulations are carried out using the guidance law (12) and Engagement 1 defined earlier. The acceleration history of the SBPN incorporating the adaptive bound estimation, in comparison with PN with $N' = 4$ and the APN, is shown in Fig. 8. Although the PN takes approximately 0.3 s longer for interception and the APN requires a slightly higher acceleration toward the end, the acceleration profiles of SBPN remain almost unchanged.

The above deterministic studies lead to the general conclusion that the performance and robustness of the PN guidance law can be significantly improved without losing its simplicity, and the SMC theory has provided the tool for achieving this objective. However, from a practical point of view, these results are to be reassessed under more representative engagement conditions by including target glint, radom aberration, and higher order seeker dynamics, along with sensor noise. Preliminary simulation results presented elsewhere[18] do reveal that the SBPN performs better than PN, even in these cases. However, the target glint significantly affects the performance of SBPN. Presently alternative forms of the switching surfaces[19] that result in a smooth guidance law and thereby reduce the effect of noise are being investigated.

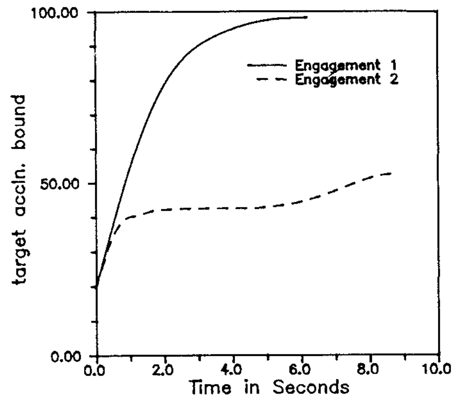


Fig. 7 Adaptive bound history.

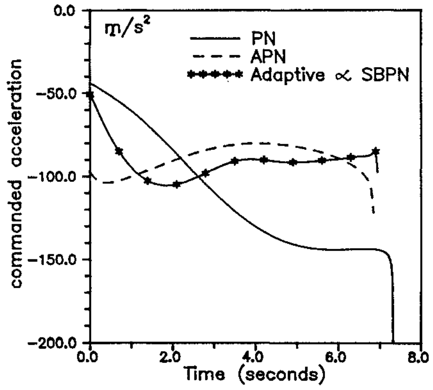


Fig. 8 Commanded acceleration profiles with decreasing missile velocity.

VI. Conclusions

In this study, a new guidance law for short-range homing missiles, based on variable structure theory, called switched bias proportional navigation, has been proposed and its performance investigated. This guidance law is structured around the basic PN with an additional switching bias term, in order to null the LOS rate to the target, even in the presence of target maneuvers. An adaptation scheme is suggested to select the gain of this term. The deterministic simulation studies involving two different engagement conditions bring out the superiority of SBPN over PN. In particular, the adaptive feature of the guidance law not only reduces the chattering associated with VSC based systems, but also helps in decreasing the magnitude of the initial transient in the commanded acceleration. SBPN has been shown to be robust with respect to the unmodeled entities like the missile velocity variation. In the ideal case, its performance approaches that of APN, where the actual target maneuver information is incorporated.

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